Partial Fractions Review

1. $\int \frac{x^2+12x-55}{(x+2)(x^2-6x+9)}\,dx \rightarrow \frac{A}{(x+2)} + \frac{B}{(x-3)} + \frac{C}{(x-3)^2}$

$x^2+12x-55 = A(x-3)^2 + B(x+2)(x-3) + C(x+2)$

$x=3$: $-10=5C$, $C=-2$

$x=-2$: $-75=25A$, $A=-3$

$A+B=1$, $B=4$

$\int \frac{-3}{x+2} + \frac{4}{x-3} + \frac{-2}{(x-3)^2}\,dx$

$u=x-3$, $du=dx$

$-2\int u^{-2}\,du$

$2u^{-1}$

$-3\ln|x+2| + 4\ln|x-3| + \frac{2}{x-3} + C$

2. $\int \frac{-2x-31}{x^2+3x-10}\,dx \rightarrow \frac{A}{(x+5)} + \frac{B}{(x-2)}$

$-2x-31 = A(x-2) + B(x+5)$

$x=2$: $-35=7B$, $B=-5$

$x=-5$: $21=-7A$, $A=-3$

$\int \frac{-3}{x+5} + \frac{-5}{x-2}\,dx$

$-3\ln|x+5| - 5\ln|x-2| + C$

3. $\int \frac{6x^3-5x^2+44x-34}{x^4+14x^2+49}\,dx \rightarrow \frac{Ax+B}{(x^2+7)} + \frac{Cx+D}{(x^2+7)^2}$

$6x^3-5x^2+44x-34 = (Ax+B)(x^2+7) + (Cx+D)$

$6x^3-5x^2+44x-34 = Ax^3+7Ax+Bx^2+7B+Cx+D$

$A=6$ $B=-5$ $7A+C=44$ $7B+D=-34$

$7(6)+C=44$ $7(-5)+D=-34$

$C=2$ $D=1$

$$\int \frac{6x}{x^2+7} + \frac{2x+1}{(x^2+7)^2}\,dx$$

$$\int \frac{6x}{x^2+7} - \frac{5}{x^2+7} + \frac{2x+1}{(x^2+7)^2}\,dx$$

$u = x^2+7$, $du = 2x\,dx$, $dx = \frac{du}{2x}$: $6\int \frac{x}{u}\cdot\frac{du}{2x}$ → $3\ln|u|$

$3\ln|x^2+7| - \frac{5}{\sqrt{7}}\arctan\frac{x}{\sqrt{7}}$

$u = x$, $a = \sqrt{7}$, $du = dx$: $-5\int \frac{1}{u^2+a^2}\,du$ → $\frac{-5}{\sqrt{7}}\arctan\frac{u}{\sqrt{7}}$

$+\int \frac{2x}{(x^2+7)^2}\,dx + \int \frac{1}{(x^2+7)^2}\,dx$

$u = x^2+7$, $\frac{du}{dx} = 2x$, $dx = \frac{du}{2x}$: $\int \frac{2x}{u^2}\cdot\frac{du}{2x}$ → $\int u^{-2}\,du$ → $-u^{-1}$

$-\frac{1}{x^2+7} + \int \frac{1}{(x^2+7)^2}\,dx$

4. $\int \frac{5x^2+19x+8}{x^3+4x^2+4x}\,dx \rightarrow \frac{A}{x} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$

$x^3+4x^2+4x = x(x+2)^2$

$5x^2+19x+8 = A(x+2)^2 + Bx(x+2) + Cx$

$x = 0$: $8 = 4A$, $A = 2$

$x = -2$: $-10 = -2C$, $C = 5$

$A + B = 5$, $B = 3$

$\int \frac{2}{x} + \frac{3}{x+2} + \frac{5}{(x+2)^2}\,dx$

$u = x+2$, $du = dx$: $5\int u^{-2}\,du$ → $-5u^{-1}$

$2\ln|x| + 3\ln|x+2| - \frac{5}{x+2} + C$

5. $\int \frac{-3x^2+4x-19}{(x-1)(x^2+5)}\,dx \rightarrow \frac{A}{x-1} + \frac{Bx+C}{x^2+5}$

$-3x^2+4x-19 = A(x^2+5) + (Bx+C)(x-1)$

$x = 1$: $-18 = 6A$, $A = -3$

$x = 0$: $-19 = -15 + -C$, $-4 = -C$, $C = 4$

$x = -1$: $-26 = -18 + (-B+4)(-2)$, $-8 = 2B-8$, $B = 0$

$$\int \frac{-3}{x-1} + \frac{4}{x^2+5}\,dx$$

$u = x \qquad a = \sqrt{5}$

$du = dx$

$$-3\ln|x-1| + \frac{4}{\sqrt{5}}\arctan\frac{x}{\sqrt{5}} + C$$

6. $$\int \frac{5x^3-3x^2+22x-5}{(x^2+4)^2}\,dx \rightarrow \frac{Ax+B}{(x^2+4)} + \frac{Cx+D}{(x^2+4)^2}$$

$$5x^3-3x^2+22x-5 = (Ax+B)(x^2+4) + Cx + D$$

$$5x^3-3x^2+22x-5 = Ax^3+4Ax+Bx^2+4B+Cx+D$$

$A = 5 \qquad B = -3 \qquad 4A + C = 22 \qquad 4B + D = -5$

$4(5) + C = 22 \qquad 4(-3) + D = -5$

$C = 2 \qquad D = 7$

$$\int \frac{5x-3}{x^2+4} + \frac{2x+7}{(x^2+4)^2}\,dx$$

$$\int \frac{5x}{x^2+4} - \frac{3}{x^2+4} + \frac{2x}{(x^2+4)^2} + \frac{7}{(x^2+4)^2}\,dx$$

$u = x^2+4 \qquad u = x \quad a = 2 \qquad u = x^2+4$

$\frac{du}{dx} = 2x \qquad du = dx \qquad \frac{du}{dx} = 2x$

$dx = \frac{du}{2x} \qquad\qquad dx = \frac{du}{2x}$

$$\int \frac{5x}{u}\cdot\frac{du}{2x} \qquad \int \frac{2x}{u^2}\cdot\frac{du}{2x}$$

$$\frac{5}{2}\ln|x^2+4| - \frac{3}{2}\arctan\frac{x}{2} - \frac{1}{x^2+4} + \int \frac{7}{(x^2+4)^2}\,dx$$